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THE FUNDAMENTAL GROUP OF WHITNEY BLOCKS

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ABSTRACT. Let X be a Peano continuum. Let $C(X)$ be the hyperspace of subcontinua of X, and let $\mu : C(X) \to \mathbf{R}$ be a Whitney map. In this paper we prove: Theorem A. If $0 \le Q < R < S < T \le \mu(X)$, then there exists a surjective homomorphism $\phi : \pi_1(\mu^{-1}(Q,R)) \to \pi_1(\mu^{-1}(S,T))$, where $\pi_1(Y)$ means the fundamental group of Y. Theorem B. If $0 \le S < T \le \mu(X)$, then $\pi_1(\mu^{-1}(S,T))$ is finitely generated. Theorem C. X is a simple closed curve if and only if $\pi_1(\mu^{-1}(S,T))$ is a nontrivial group for every $0 \le S < T \le \mu(X)$.

0. Introduction. Throughout this paper X will denote a continuum (a nonempty, compact, connected metric space) with metric $\boldsymbol{d}$. Let $C(X)$ denote the hyperspace of all subcontinua of X with the Hausdorff metric $\mathcal{H}$. A map is a continuous function. A *Whitney map* for $C(X)$ is a map $\mu : C(X) \to \mathbf{R}$ such that (a) $\mu(\{x\}) = 0$ for every $x \in X$, (b) If $A, B \in C(X)$ and $A \subset B \neq A$, then $\mu(A) < \mu(B)$, and (c) $\mu(X) = 1$. A *Whitney block* for $C(X)$, respectively a *Whitney level* for $C(X)$, is a set of the form $\mu^{-1}(S,T)$, respectively $\mu^{-1}(T)$, where $0 \le S < T \le 1$. The fundamental group of a space Y is denoted by $\pi_1(Y)$.

Hyperspaces are acyclic (see [**13**, Theorem 1.2]). For Whitney levels, the situation is different; the following observation was made by J.T. Rogers, Jr., in [**11**]: "As we go higher into the hyperspace, no new one-dimensional holes are created, and perhaps some one-dimensional holes are swallowed." This intuitive statement has found several formulations.

In [**12**, Theorem 5], J.T. Rogers, Jr., proved:

Theorem. *If μ is a Whitney map for $C(X)$ and $0 \le s \le t \le 1$, then there exists a monomorphism*

$$\gamma^* : H^1(\mu^{-1}(t)) \longrightarrow H^1(\mu^{-1}(s))$$

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