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ALMOST RECONSTRUCTION OF THE 3-DIMENSIONAL BALL FROM $K_{pqrs} \times I$

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ABSTRACT. Let K_{pqrs} denote the 2-complex obtained by attaching two disks to the wedge of two circles by the words $a^p b^q$ and $a^r b^s$, with $ps - qr = \pm 1$. The complex K_{pqrs} is contractible. If the Zeeman conjecture is true, then $K_{pqrs} \times I$ is collapsible. This paper proves that $K_{pqrs} \times I$ collapses to a 2-sphere S^2 plus its interior. The proof exhibits K_{pqrs} as the spine of a 3-ball under a retraction map whose restriction to the boundary S^2 lifts to an embedding into $K_{pqrs} \times I$.

1. Introduction. Let K_{pqrs} denote the 2-complex obtained by attaching two disks to the wedge of two circles by the words $a^p b^q$ and $a^r b^s$, with $ps - qr = \pm 1$. The set K_{pqrs} is contractible. Let I denote a unit interval. In [**3**], Zeeman posed the question: Is $K_{pqrs} \times I$ collapsible? Zeeman proved that the answer is affirmative for K_{1112}, the "topological dunce hat." In [**2**], Lickorish provided an affirmative answer for K_{2334}, but observed that his methodology did not seem to generalize to all K_{pqrs}. In this paper, we display a structure on all K_{pqrs} which seems to correspond to the initial phase of Lickorish's collapse of $K_{2334} \times I$, and which is valid for all K_{pqrs}: The product $K_{pqrs} \times I$ collapses to a 2-*sphere plus its interior*.

Definition. Let S^2 denote a piecewise linear 2-sphere in a contractible 3-complex K^3. A point p of $K^3 - S^2$ lies in the *interior* of S^2 if S^2 is nontrivial in $H_2(K^3 - p; Z_2)$, the second homology group of $K^3 - p$ with Z_2-coefficients.

This definition is commonly used in the special case of a 2-sphere plus its interior in Euclidean 3-space (the three-dimensional PL Schoenflies theorem). In this paper we concern ourselves with constructing a 2-sphere plus its interior in the set $K_{pqrs} \times I$.

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