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# SUBGROUP SEPARABILITY OF CERTAIN HNN EXTENSIONS OF FINITELY GENERATED ABELIAN GROUPS

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ABSTRACT. In this note we give a characterization for certain HNN extensions of finitely generated abelian groups to be subgroup separable.

**1.** A group $G$ is called subgroup separable if, for each finitely generated subgroup $M$ and for each $x \in G\backslash M$, there exists a normal subgroup $N$ of finite index in $G$ such that $x \notin MN$. It is well known that free groups and polycyclic groups (and hence finitely generated abelian groups) are subgroup separable [**6**, **7**].

Let $G = \langle t, K; t^{-1}At = B, \varphi\rangle$ be an HNN extension where $K$ is a finitely generated abelian group, $A$ and $B$ are the associated subgroups and $\varphi$ is the associated isomorphism $\varphi : A \to B$. Suppose $A$ and $B$ have finite index in $K$. In this note we give a characterization for the HNN extension $G$ to be subgroup separable. We shall prove the following:

**Theorem 1.** *Let* $G = \langle t, K; t^{-1}At = B, \varphi\rangle$ *be an* HNN *extension where* $K$ *is a finitely generated abelian group and* $A$ *and* $B$ *have finite index in* $K$. *Then the following are equivalent*:

(i) $G$ *is subgroup separable*;

(ii) *Either* $K = A = B$ *or there exists a subgroup* $H$ *of finite index in* $K$ *and* $H$ *is normal in* $G$;

(iii) *There exists a finitely generated abelian group* $X$ *such that* $K$ *is a subgroup of finite index in* $X$ *and an automorphism* $\bar{\varphi} \in \operatorname{Aut} X$ *with* $\bar{\varphi}|_A = \varphi$.

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