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VARIATIONAL METHODS AND SPECTRAL ASYMPTOTICS OF TWO PARAMETER ELLIPTIC EIGENVALUE PROBLEMS IN A BALL

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ABSTRACT. We consider the following nonlinear two-parameter elliptic eigenvalue problem

$$\begin{cases} \Delta u + \mu u^p = \lambda u & \text{in } B = \{x \in R^N : |x| < 1\}, \\ u > 0 & \text{in } B, \\ u = 0 & \text{on } \partial B, \end{cases}$$

where $N \geq 2$, $p > 1$ and $\mu, \lambda > 0$ are eigenvalue parameters. We apply two different kind of variational methods to this problem and define the variational eigenvalues $\lambda = \lambda(\mu)$ and $\mu = \mu(\lambda)$. Then we shall establish the asymptotic formulas of $\lambda(\mu)$ and $\mu(\lambda)$ as $\mu \to \infty$ and $\lambda \to \infty$, respectively, and the close relationship between the two asymptotic formulas are confirmed.

1. Introduction. We consider the following nonlinear two-parameter elliptic eigenvalue problems in a ball:

$$\begin{cases} \Delta u + \mu u^p = \lambda u & \text{in } B = \{x \in R^N : |x| < 1\}, \\ u > 0 & \text{in } B, \\ u = 0 & \text{on } \partial B, \end{cases} \tag{1.1}$$

where $N \geq 2$, $p > 1$ and $\mu, \lambda \in R$ are eigenvalue parameters.

Since all positive solutions of (1.1) are radially symmetric (cf. Gidas, Ni and Nirenberg [**3**]), we consider the ordinary differential equation

$$\begin{cases} u''(r) + \frac{N-1}{r} u'(r) + \mu u^p = \lambda u, & 0 < r < 1, \\ u(r) > 0 & 0 \leq r < 1, \\ u'(0) = 0, \qquad u(1) = 0, \end{cases} \tag{1.2}$$

where $r = |x|$.

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