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A REVERSED MEIR'S INEQUALITY AND SOME RELATED RESULTS

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ABSTRACT. Reversed versions are developed for Meir's inequality and some related results.

1. Introduction. In 1981 Meir [**4**] proved the following theorem for nondecreasing sequences.

Theorem A. *Let* $a_0, a_1, \dots, a_{n-1}$ *and* $p_1, p_2, \dots, p_n$ *be nonnegative real numbers satisfying*

$$0 = a_0 \le a_1 \le a_2 \le \cdots \le a_{n-1}, \tag{1.1}$$

$$a_i - a_{i-1} \le p_i, \qquad i = 1, \dots, n-1, \tag{1.2}$$

and

$$p_1 \le p_2 \le \cdots \le p_n. \tag{1.3}$$

If r *and* s *are real numbers with* $r \ge 1$ *and* $s \ge 2r+1$, *then*

$$\left[(s+1)\sum_{i=1}^{n-1} a_i^s (p_i + p_{i+1})/2\right]^{1/(s+1)} \le \left[(r+1)\sum_{i=1}^{n-1} a_i^r (p_i + p_{i+1})/2\right]^{1/(r+1)}. \tag{1.4}$$

Theorem A is sufficiently complicated for its history to be worth noting. This began with Klamkin and Newman [**3**] noting in 1976 that the striking elementary identity

$$\sum_{j=1}^{n} j^3 = \left[\sum_{j=1}^{n} j\right]^2$$

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