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MOMENT PROBLEM FOR RATIONAL ORTHOGONAL FUNCTIONS ON THE UNIT CIRCLE

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ABSTRACT. The Favard-type theorem for rational functions orthogonal on the unit circle with prescribed poles lying outside the unit circle are studied. We also consider the existence of sequences of orthogonal rational functions whose zeros are everywhere dense in $|z| \leq 1$.

1. Introduction. Let $d\mu$ be a finite positive Borel measure with an infinite set as its support on $[0, 2\pi)$. We define $L^2_{d\mu}$ to be the space of all functions $f(z)$ on the unit circle $T := \{z \in \mathbf{C} : |z| = 1\}$ satisfying $\int_0^{2\pi} |f(e^{i\theta})|^2 \, d\mu(\theta) < \infty$. Then $L^2_{d\mu}$ is a Hilbert space with inner product

$$\langle f, g \rangle := \frac{1}{2\pi} \int_0^{2\pi} f(e^{i\theta}) \overline{g(e^{i\theta})} \, d\mu(\theta).$$

Consider a sequence $\{z_n\}$ with $|z_n| < 1$, and let

$$b_n(z) := \frac{z_n - z}{1 - \bar{z}_n z} \eta_n \quad \text{and} \quad \eta_n := \frac{|z_n|}{z_n}, \quad n = 1, \dots ,$$

where for $z_n = 0$ we put $|z_n|/z_n = -1$. Next we define finite Blaschke products recursively as

$$B_0(z) = 1 \quad \text{and} \quad B_n(z) = B_{n-1}(z) b_n(z), \quad n = 1, \dots .$$

The fundamental polynomials $w_n(z)$ are given by

$$w_0(z) := 1 \quad \text{and} \quad w_n(z) := \prod_{i=1}^{n} (1 - \bar{z}_i z), \quad n = 1, \dots .$$

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