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GEOMETRY OF BANACH SPACES WITH (α, ε)-PROPERTY OR (β, ε)-PROPERTY

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ABSTRACT. Many authors investigated Banach spaces with property α or property β. They showed that a space with property α (property β) shares many geometrical properties of $l_1(l_\infty)$. We shall investigate the structure of the unit sphere of Banach spaces with property α or β in terms of points of local uniform rotundity, Fréchet differentiability and vertex points. As a consequence of this, we obtain that every Banach space can be renormed in such a way that there is no locally uniformly rotund point but the set of points of Fréchet differentiability for the norm is an open and norm dense subset of the space.

1. Introduction. Properties A and B were defined by J. Lindenstrauss [**7**] in the study of norm attaining operators. The Banach space X has property A if, for every Banach space Y, the norm attaining operators are dense in $L(X, Y)$ and the Banach space Y has property B if, for every Banach space X, the norm attaining operators are dense in $L(X, Y)$. He gave two geometric criteria for property A and B named property α and β [**7, 13**]. J. Partington [**12**] proved that every Banach space can be $(3+\varepsilon)$-equivalently renormed to have property β but, if the continuum hypothesis is assumed, a nonseparable Banach space is constructed in [**11**] which cannot be equivalently renormed to have property α. As a consequence, we observe that not every dual Banach space admits an equivalent dual norm with property β. Properties α and β generalize, in some sense, the geometric situation of l_1 and l_∞, as is pointed out in [**3, 5, 6** and **13**].

A *vertex* point of a closed bounded convex body C is a point which is strongly exposed by an open set of functionals. A *face* is the

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