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ASYMPTOTIC BEHAVIOR OF SOLUTIONS OF THREE-TERM POINCARÉ DIFFERENCE EQUATIONS

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ABSTRACT. Let $\{X_n\}$ be a solution of the difference equation

$$X_n = (b + \delta_n)X_{n-1} + (a + \varepsilon_n)X_{n-2}.$$

Then it is well known that the limit $\lim X_n/(b+x)^n$ exists (and is finite) if $\sum |\varepsilon_n| < \infty$, $\sum |\delta_n| < \infty$ and $x(b + x) = a$ with $|x| < |b + x|$. We generalize this result to cases where $\sum \varepsilon_n$ and $\sum \delta_n$ only converge conditionally. Such results have applications to the study of asymptotic behavior of orthogonal polynomials and orthogonal functions, and to the study of separate convergence of continued fractions.

1. Introduction. We shall study solutions $\{Z_n\}_{n=-1}^{\infty}$ of the Poincaré difference equation

$$Z_n = b_n Z_{n-1} + a_n Z_{n-2}; \qquad n = 1, 2, 3, \dots ; \ a_n \neq 0, \tag{1.1}$$

where a_n and b_n are either complex numbers or complex valued functions such that

$$\begin{gathered} a_n \longrightarrow a \in \mathbf{C}, \quad b_n \longrightarrow b \in \mathbf{C}\backslash\{0\}, \\ a/b^2 \in \mathbf{C}\backslash(-\infty, -1/4]. \end{gathered} \tag{1.2}$$

The characteristic equation of (1.1), $\lambda^2 = b\lambda + a$, has two solutions $x_1 = -x$ and $x_2 = b + x$, where

$$x := b(\sqrt{1 + 4a/b^2} - 1)/2; \qquad \Re\sqrt{1 + 4a/b^2} > 0, \tag{1.3}$$

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