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# TRAVELLING FRONTS IN CYLINDERS AND THEIR STABILITY

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*Dedicated to Louis Nirenberg on the occasion of his 70th birthday*

**1. Introduction.** We study the qualitative properties of travelling fronts of the semilinear parabolic equation

$$\partial u/\partial t - \Delta u = f(u), \quad (t,x) \in (0,\infty) \times \Sigma,$$

where $\Sigma = \mathbf{R} \times \Omega$ with $\Omega \subset \mathbf{R}^{n-1}$ being a bounded smooth domain and $n \geq 2$. We often denote $x \in \Sigma$ by $x = (x_1, y)$ with $x_1 \in \mathbf{R}$ and $y \in \Omega$, and the outer unit normal to $\partial\Omega$ or to $\partial\Sigma$ by $\nu$.

In the above equation the term $f(u)$ represents a source term with $f(0) = f(1) = 0$. Equations as above have been derived to model problems arising from applied sciences, such as population dynamics, genetics, combustion and flame propagation. In these situations one of the most interesting and natural questions is the behavior of solutions $u(x,t)$ as $t \to +\infty$; in particular, the question about the existence of travelling fronts, and whether the general solutions approach travelling fronts (i.e., the stability of travelling fronts). Travelling fronts are solutions of the form $u = u(x_1+ct, y)$ satisfying $u \to 0$ or 1 as $x_1 \to -\infty$ or $+\infty$, respectively (here $c$ is a real constant and is usually referred to as the speed of the front). In the past several decades, the questions of existence, nonexistence and stability of travelling fronts have attracted the attention of many mathematicians, leading to the production of a large literature on the subject. The interested readers may refer to the book [**17**] by P. Fife, the paper [**14**] by H. Berestycki and L. Nirenberg and the papers [**37, 38**] by A. Volpert (see also, [**1, 2, 4**–**13**, **15**–**26**, **29**–**36**, **39**–**42**]) for the history of problems related to travelling fronts.

Throughout this paper the homogeneous Neumann boundary conditions on $\partial\Sigma$ are assumed. Therefore, the following equation must be

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