

ON THE MINIMAL FREE RESOLUTION
OF GENERAL k -GONAL CURVES

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ABSTRACT. Here we study the minimal free resolution of general embeddings of a curve in \mathbf{P}^n ; the curve may be a general k -gonal curve and the embedding linearly normal (the aim is to prove condition N_p for suitable degree, genus and p) or not linearly normal (for low p the embedded curve has the same type of minimal free resolution).

0. Introduction. The main aim of this paper is to show that (as remarked in [5]) the proofs in [5] may be used to obtain several other nontrivial results. The topic is the minimal free resolution of a closed subscheme Z of \mathbf{P}^n . Here (as in [5]) we consider the case $\dim(Z) = 1$. We stress that [5] owes very much to [11]. To state our results we fix some notations and definitions. Let Z be a closed subscheme of \mathbf{P}^n , and let

$$(1) \quad \cdots \longrightarrow E_{i+1} \longrightarrow E_i \longrightarrow \cdots \longrightarrow E_1 \longrightarrow \mathbf{I}_Z \longrightarrow 0$$

be the minimal free resolution of the ideal sheaf of Z . By definition this means that (1) is an exact sequence of sheaves with each E_i direct sum of line bundles, with $E_j = 0$ for $j > n$ and such that for every i if a line bundle, L , is a direct summand both in E_{i+1} and E_i the map $L \rightarrow L$ induced by the map $E_{i+1} \rightarrow E_i$ is zero.

Definition 0.1. Let $C \subset \mathbf{P}^n$ be a reduced curve; fix an integer $p \geq 1$. We will say that C satisfies the property N_p if C is arithmetically Cohen-Macaulay and for every integer i with $1 \leq i \leq p$ the i th-sheaf appearing in the minimal free resolution of the homogeneous ideal of C is the direct sum of line bundles of degree $-i - 1$.

For instance, if we say that N_0 means C is *arithmetically Cohen-Macaulay*, then N_1 means that the curve C is N_0 and its homogeneous

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