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DISTRIBUTIONAL CONTROL FOR OPERATORS ON VECTOR-VALUED L^p-SPACES

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1. Introduction. Throughout this paper, $(\Omega, \mathcal{F}, \mu)$ will denote an arbitrary measure space, and X will be an arbitrary Banach space with norm denoted $\|\cdot\|$. A function $f : \Omega \to X$ is said to be strongly measurable if there exists a sequence $\{f_n\}$ of X-valued simple functions on Ω such that $\lim_{n\to\infty} f_n(\omega) = f(\omega)$ μ almost everywhere on Ω. For each $p \in [1, \infty)$, let $L^p(\Omega, \mu, X)$ denote the set of all strongly measurable functions which satisfy $\int_\Omega \|f(\omega)\|^p \, d\mu(\omega) < \infty$. Identifying functions that are equal μ-almost everywhere, this is a Banach space with norm $\|f\|_p = (\int_\Omega \|f(\omega)\|^p \, d\mu(\omega))^{1/p}$. Similarly, the set of essentially bounded, strongly measurable functions from Ω into X, after identifying functions which are equal μ almost everywhere, becomes a Banach space with norm $\|f\|_\infty = \text{ess sup}\{\|f(\omega)\| : \omega \in \Omega\}$ and is denoted by $L^\infty(\Omega, \mu, X)$. For all $p \in [1, \infty]$, $L^p(\Omega, \mu, X)$ will also be denoted by E^p. A function $f : \Omega \to X$ is called weakly measurable if for each $x^* \in X^*$, $t \mapsto x^*(f(t))$ is a measurable scalar-valued function.

In Section 2 we define distributional control for operators and representations. We show that an operator T that is distributionally controlled can be extended to an operator $T^{(p)}$ on the norm-closure of $E^1 \cap E^\infty$ in E^p for $1 \le p \le \infty$. We obtain structural information for such operators that is motivated by the scalar-valued results in [**2**]. With these results, we construct a distributionally controlled operator on $L^1(\Omega) \cap L^\infty(\Omega)$ that dominates T. This concept is related to the linear modulus of an operator as introduced in [**3**], and the L^p-majorant for operators on vector-valued L^p-spaces as introduced in [**4**].

In the case of scalar-valued L^p-spaces, representations consisting of distributionally controlled operators have been used to derive general ergodic theorems (see [**1**] and [**2**] for illustrations). With this in mind, we have specialized some of our results to representations of locally

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