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ON SOME QUASILINEAR SYSTEMS

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0. Introduction. In this paper we will be interested in systems of quasilinear equations whose action-functional is strongly indefinite. The general problem is the following

$$\text{(P)} \qquad \begin{cases} -\Delta_p u = F_u(u,v) & \text{in } \Omega \\ \Delta_q v = F_v(u,v) & \text{in } \Omega \\ u = v = 0 & \text{on } \partial\Omega \end{cases}$$

where $\Omega \subset \mathbf{R}^N$ is a bounded domain with smooth boundary, $N > 2$, $1 < p,\ q < N$ and F is a $\mathcal{C}^1$-function. On the function F we impose the following

Coupling condition (C). *The equations $F_u(0,v) = 0$ and $F_v(u,0) = 0$ have only finitely many solutions.*

We shall see later on that, due to this coupling condition, it is impossible for the Problem (P) to have solutions of the form $(u,0)$ or $(0,v)$.

Our goal is to give two methods for finding solutions of Problem (P). Section 1 will be devoted to studying Problem (P) by means of the reduction to a semilinear system following the ideas by [**8, 9**]. Here the method consists of a convenient splitting and using a duality argument (see [**7**] and [**5**]). The remaining sections will be dealing with a more general case using a Galerkin type argument in combination with the finite dimensional linking theorem (see [**1, 14**]). The Galerkin approach was previously used by other authors [**3, 11, 14**].

1. The splitting method. Due to technical obstructions, we study a particular case where we assume $2N/(N+2) < p < 2$, $q = 2$. Notice that the equations of Problem (P) are the Euler Lagrange equations of

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