ROCKY MOUNTAIN
JOURNAL OF MATHEMATICS
Volume 27, Number 3, Summer 1997

ON NEW MAJORIZATION THEOREMS

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ABSTRACT. The subject of majorization is treated extensively, see, for instance, [**1, 5**] and [**4**] and their references.

In 1947, L. Fuchs gave a weighted generalization of the well-known majorization theorem for convex functions and two sequences monotonic in the same sense, see [**4**, p. 419] or [**6**, p. 323]. For other related results, see [**4**, pp. 417–420] or [**6**, pp. 323–332].

In this paper we shall give related results in the case when only one sequence is monotonic. Moreover, while in Fuchs results we have real weights, in our results we need positive weights. These results form extensions of theorems of Kolumban and Mocanu [**2**], Toader [**8**] and Maligranda, Pečarić and Persson [**3**], as well as results from [**5**, pp. 337–338; 350] and [**7**, pp. 9 and 145–148].

1. Main results.

Theorem 1. *Let g be a strictly increasing function from (a,b) to (c,d), and let $f\circ g^{-1}$ be a concave function on $[c,d]$. Let the vectors $\mathbf{x}$ and $\mathbf{y}$ with elements from (a,b) satisfy*

$$\sum_{i=1}^k w_i g(x_i) \geq \sum_{i=1}^k w_i g(y_i), \quad k=1,\dots,n. \tag{1.1}$$

(a) *If*

(a_1) *f is decreasing*

(a_2) *the vector $\mathbf{y}$ is decreasing, then*

$$\sum_{i=1}^k w_i f(x_i) \leq \sum_{i=1}^k w_i f(y_i), \quad k=1,\dots,n. \tag{1.2}$$

Received by the editors on October 10, 1994, and in revised form on March 25, 1997.