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TRANSITION VECTOR MEASURES AND MULTIMEASURES AND PARAMETRIC SET-VALUED INTEGRALS

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ABSTRACT. In this paper we examine transition vector measures and multimeasures. First we prove a Radon-Nikodym theorem for transition vector measures and then we use it to establish the existence of a set-valued Radon-Nikodym derivative for transition multimeasures. Subsequently, we examine parametric set-valued integrals and obtain two results characterizing their measurable selectors (integral versions of Filippov's implicit function lemma). We conclude with a useful observation concerning transition measures.

1. Introduction. In a recent paper [**10**] we proved a Radon-Nikodym theorem for transition multimeasures (set-valued measures). The purpose of this paper is to extend the above mentioned results to the case where the dominating (control) measure is a transition measure too and then establish some properties of parametric set-valued integrals. Multimeasures and set-valued integrals are the natural generalization of classical single-valued measures and integrals and so it is interesting to know to what extent we can duplicate the existing theory on them. But, in addition, multimeasures and set-valued integrals are the appropriate analytical tools in various applied areas, like mathematical economics, statistics, optimization and optimal control. We refer to [**10, 11**] and [**12**] for a list of relevant references. We also mention that transition multimeasures are useful in the study of Markov temporary equilibrium processes in dynamic economies; see Blume [**2**].

In this paper, first we prove a Radon-Nikodym theorem for transition vector valued measures and then we use that result to prove a Radon-Nikodym theorem for transition multimeasures. Then, in Section 4,

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