ROCKY MOUNTAIN
JOURNAL OF MATHEMATICS
Volume 27, Number 3, Summer 1997

AN EQUIVALENCE FOR CATEGORIES OF MODULES OVER A COMPLETE DISCRETE VALUATION DOMAIN

PATRICK KEEF

ABSTRACT. An equivalence is presented between two categories closely related to the category of valuated vector spaces. This equivalence is then recast in terms of short exact sequences of modules. A new class of mixed modules is introduced, whose members we call $\mathcal{A}$-*Warfield*, and, in particular, it is shown that an isomorphism between the endomorphism rings of two $\mathcal{A}$-Warfield modules is induced by an isomorphism of their underlying modules.

1. Introduction. Valuated vector spaces have proven to be of great importance in the study of abelian groups, see [**2**]. For example, if G is any abelian p-group, its socle $G[p]$ inherits a valuation from the height function on G. The main purpose of this note is to explore two closely related constructions using other well-behaved classes of algebraic objects. Let R be a fixed complete discrete valuation domain with quotient ring Q and $p \in R$ prime.

Define a category $\mathcal{A}$ as follows. An object $A \in \mathcal{A}$ is a reduced torsion-free algebraically compact R-module, together with a smoothly descending chain of summands $A[\alpha]$ indexed by the ordinals (where we consider the symbol ∞ to be an ordinal greater than all other conventional ordinals), starting with $A[0] = A$. It follows that, for every α, $A[\alpha]$ is also reduced, torsion-free and algebraically compact. Notice that, if $A[\alpha]$ is defined only for isolated ordinals and for each limit ordinal λ we define $A[\lambda] = \cap_{\alpha<\lambda} A[\alpha]$, then $A[\lambda]$ is pure (since A is torsion-free) and p-adically closed, hence it is also a summand of A.

Define a second category $\mathcal{D}$ as follows. An object $D \in \mathcal{D}$ is a divisible torsion R-module, together with a smoothly descending chain of submodules $D[\alpha]$ indexed by the ordinals, starting with $D[0] = D$, such that $D[\alpha]$ is a summand of D whenever α is isolated. Once again, to specify the chain of submodules, it is only necessary to describe

Received by the editors on December 16, 1994, and in revised form on July 17, 1995.
Research supported in part by NSF (RUI) Grant DMS-9405118.