ROCKY MOUNTAIN
JOURNAL OF MATHEMATICS
Volume 27, Number 3, Summer 1997

SINGULAR POINTS OF ANALYTIC FUNCTIONS EXPANDED IN SERIES OF FABER POLYNOMIALS

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ABSTRACT. Let $a_n \geq 0$, $n = 0, 1, \ldots$, be such that $\limsup_{n\to\infty}(a_n)^{1/n} = 1$. Then a theorem of Pringsheim states that the point $z = 1$ is a singular point for $f(z) = \sum_{n=0}^{\infty} a_n z^n$. It is the purpose of this note to extend Pringsheim's theorem by replacing the unit disk $|z| \leq 1$ by a compact simply connected set E (containing more than one point) and whose boundary $\mathrm{Br}\,(E)$ is an analytic Jordan curve, and by replacing the monomials z^n by the Faber polynomials for E.

1. Introduction. Let $a_n \geq 0$, $n = 0, 1, \ldots$, be such that

$$\limsup_{n\to\infty}(a_n)^{1/n} = 1. \tag{1.1}$$

Then a theorem of Pringsheim [**8**] states that the point $z = 1$ is a singular point for

$$f(z) = \sum_{n=0}^{\infty} a_n z^n.$$

It is the purpose of this note to extend Pringsheim's theorem by replacing the unit disk $|z| \leq 1$ by a compact simple connected set E (containing more than one point) and whose boundary $\mathrm{Br}\,(E)$ is an analytic Jordan curve, and by replacing the monomials z^n by the Faber polynomials for E.

For the sake of notational simplicity we will assume that the capacity of E, $\mathrm{Cap}\,(E)$, is equal to 1. It will appear clearly, however, that our results hold for any positive value of $\mathrm{Cap}\,(E)$.

The function $w = \phi(z)$ which maps conformally the exterior of E, $\mathrm{Ext}\,(E)$ onto $|w| > 1$ and such that $\phi(\infty) = \infty$, has a Laurent expansion at infinity of the form

$$\phi(z) = z + a_0 + \frac{\alpha_{-1}}{z} + \cdots .$$

Received by the editors on June 8, 1995.
1991 *Mathematics Subject Classification.* 30B40.