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UNIQUENESS OF SMALL SOLUTIONS TO THE DIRICHLET PROBLEM FOR THE HIGHER DIMENSIONAL H-SYSTEM

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ABSTRACT. Let $\Omega \subset \mathbf{R}^n$, $n \geq 2$, $u, v : \Omega \to \mathbf{R}^{n+1}$ be two solutions of the constant mean curvature equation

$$\sum_{i=1}^{n} \frac{\partial}{\partial x_i}\left(|\nabla u|^{n-2}\frac{\partial}{\partial x_i}u\right) = n^{n/2}H(u_{x_1} \times \cdots \times u_{x_n}).$$

Assume $u = v$ on $\partial\Omega$ and $|H| \max(\sup_\Omega |u|, \sup_\Omega |v|) < 1/n$. Then u and v coincide in Ω.

The Dirichlet problem for the equation of surfaces of prescribed mean curvature in $\mathbf{R}^3$

$$\Delta u = 2H(u_{x_1} \times u_{x_2})$$

has been frequently studied, and great progress has been achieved in the last decades. We only mention the important contributions of E. Heinz [**3, 4**], S. Hildebrandt [**5**] and H.C. Wente [**9**]. A far more extensive bibliography can, e.g., be found in [**1**]. Under reasonable assumptions on H and the prescribed boundary values, existence, uniqueness and regularity of "small" solutions have been proven.

F. Duzaar and M. Fuchs [**1, 2**] have studied the higher dimensional analogue

$$\sum_{i=1}^{n} \frac{\partial}{\partial x_i}\left(|\nabla u|^{n-2}\frac{\partial}{\partial x_i}u\right) = n^{n/2}H(u_{x_1} \times \cdots \times u_{x_n}) \quad \text{in } \Omega \subset \mathbf{R}^n, \qquad u = u_0 \quad \text{on } \partial\Omega. \tag{1}$$

Here $\Omega \subset \mathbf{R}^n$, $n \geq 2$, is a smoothly bounded domain, $u : \overline{\Omega} \to \mathbf{R}^{n+1}$ is the unknown vector function, H is a constant and u_0 a sufficiently

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