ROCKY MOUNTAIN
JOURNAL OF MATHEMATICS
Volume 27, Number 3, Summer 1997

A BOUNDARY VALUE PROBLEM FOR A SYSTEM OF ORDINARY DIFFERENTIAL EQUATIONS WITH IMPULSE EFFECTS

PAUL W. ELOE AND JOHNNY HENDERSON

ABSTRACT. A two-point boundary value problem for a system of first order ordinary differential equations with impulse effects is studied. The method of upper and lower solutions is employed to obtain the existence of a solution and a method of forced monotonicity is employed to obtain iterative improvement. The main result is illustrated with an application to the Liénard equation with periodic boundary conditions.

1. Introduction. Let $n \geq 1$, $m \geq 0$ be integers. Let $I = [a, b] \subset \mathbf{R}$, and let $a = t_0 < t_1 < \cdots < t_{m+1} = b$ be given. Let $f : I \times \mathbf{R}^n \to \mathbf{R}^n$, $r_k : I \times \mathbf{R}^n \to \mathbf{R}^n$, $k = 1, \dots, m$, be continuous. Let M and N be $n \times n$ matrices with real entries, and let $c \in \mathbf{R}^n$. We shall study the impulsive boundary value problem (BVP) for the system of first order differential equations,

$$y' = f(t, y), \quad t \in I \backslash \{t_1, \dots, t_m\}, \tag{1.1}$$

$$\Delta y(t_k) = r_k(t_k, y(t_k^-)), \quad k = 1, \dots, m, \tag{1.2}$$

$$My(a) + Ny(b) = c, \tag{1.3}$$

where $\Delta y(t) = y(t^+) - y(t^-)$. For simplicity, we shall sometimes denote $y(t^-)$ by $y(t)$ and we shall sometimes denote the boundary operator, $My(a) + Ny(b)$, by Ty; note that we shall consider an impulsive BVP with fixed moments.

Bainov et al. [**2, 3, 4**] have developed the theory of impulsive differential equations. An extensive literature exists and is documented in [**3**]. In the case of periodic systems, Bainov and Simeonov [**3**] have

Received by the editors on August 5, 1994.
1991 *Mathematics Subject Classification.* Primary 34B15, Secondary 34A45.