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COMPLEMENTED COPIES OF l_1 IN $L^\infty(\mu, X)$

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An active field of research in recent years has been the study of the inclusion, as a subspace or complemented subspace, of classical Banach sequence spaces such as c_0, l_1 or l_∞ in Banach spaces $L^p(\mu, X)$ of Bochner p-integrable (essentially bounded for $p = \infty$) functions over a finite measure space (Ω, Σ, μ) with values in a Banach space X. The following problem, originally posed by Labuda, is mentioned in [**4**, p. 389]: When does $L^\infty(\mu, X)$ contain a complemented copy of l_1? Natural conjectures such as "if (and only if) X has a (complemented) copy of l_1," were disproved by an example due to Montgomery-Smith [**4**, p. 389]: there is a Banach space X with separable dual such that $L^\infty(\mu, X)$ contains a complemented copy of l_1. The aim of this paper is to answer this question for the case when X is a Banach lattice.

Theorem. *Let X be a Banach lattice. The following are equivalent*:

(1) $L^\infty(\mu, X)$ *contains a complemented subspace isomorphic to* $L^1[0, 1]$.

(2) $L^\infty(\mu, X)$ *contains a complemented subspace isomorphic to* l_1.

(3) $l_\infty(X)$ *contains all* l_1^n *uniformly complemented.*

(4) X *contains all* l_1^n *uniformly complemented.*

Before proving this theorem, let us recall a few notions from the local theory of Banach spaces. A normed space X is said to be an $\mathcal{S}_p$-space, $1 \le p \le \infty$, if it contains all l_p^n uniformly complemented, i.e., if there is some $\lambda \ge 1$ such that, for every $n \in \mathbf{N}$ there are operators $J_n \in L(l_p^n, X)$ and $P_n \in L(X, l_p^n)$, satisfying

$$P_n J_n = \mathrm{id}_{l_p^n}; \qquad \|P_n\|\,\|J_n\| \le \lambda.$$

We may assume throughout that $\|P_n\| \le \lambda$ and $\|J_n\| \le 1$, for all $n \in \mathbf{N}$.

The terminology an notations are standard except, perhaps, the following one: if (A_n) is a sequence of pairwise disjoint measurable

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