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A NOTE ON PRIME n-TUPLES

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Twin primes and *prime triples* are common names given to special prime numbers related to a famous conjecture of Goldbach. A twin prime is an integer p such that p and $p+2$ are both prime numbers. The so-called "twin prime conjecture" states that there exist infinitely many twin primes. Although believed to be true, it remains an intriguing open question.

Prime triples with respect to two integers $\{r, s\}$ are integers p such that $p, p+r$ and $p+s$ are all primes. The question of how many prime triples exist with respect to a given $\{r, s\}$ depends, very much so, on r and s. Only two prime triples with respect to $\{2, 4\}$ exist, namely, $p = 1$ and $p = 3$, whereas, for the case $r = 2$ and $s = 6$ it is again a widely open question. In [**3**, Problem 4, p. 177] a bound on $\pi_3(x)$, the number of all prime triples with respect to $\{2, 6\}$ that are less than x is given. Here, too, infinitely many prime triples of these are believed to exist.

Prime n-tuples with respect to $\{r_1, r_2, \dots, r_{n-1}\}$ are similarly defined and for an intelligent guess of the r_i's, i.e., where one cannot prove by elementary means that there are finitely many prime n-tuples, the conjecture is that there are infinitely many such primes. Clearly, consecutive prime n-tuples are farther apart as n gets larger. The following theorem gives a partial quantitative measure of that spread. To the best of our knowledge our method is new.

Theorem. *Let $Q = \{q_1 < q_2 < \dots < q_n\}$ and $P = \{p_1 < p_2 < \dots < p_n\}$ be two sets of positive integers such that each p_i is a prime $(n+1)$-tuple with respect to Q. Then there exists a positive constant c, independent of n, such that*

$$(p_n - p_1)(q_n - q_1) \geq cn^4.$$

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