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A CLASS OF CAUCHY PROBLEMS THAT INVOLVE FACTORABLE DIFFERENTIAL OPERATORS

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ABSTRACT. Let $x = (x_1, \dots, x_p)$, $D = (D_1, \dots, D_p)$, and let $P(D)$ be a linear partial differential operator with constant coefficients that can be factored over the complex field into a product of linear combinations of the D_j. Using the simple quasi inner product (qip), we obtain representations of solutions of a class of Cauchy problems that includes $\partial^n w(x,t)/\partial t^n = P(D)w(x,t)$, $\partial^j w(x,t)/\partial t^j|_{t=0} = \phi_j(x)$ for $j = 0, 1, \dots, n-1$ as multiple integrals of complex translations of the data functions. The factor switching property of the qip plays a central role in constructing these representations and imposing smoothness restrictions on the data. Examples are given to illustrate the flexibility that the qip permits in altering solution forms to fit in with the data or in determining optimal growth conditions for entire data.

1. Introduction. Let p be a positive integer with $p \geq 2$, let $x = (x_1, x_2, \dots, x_p)$, and let $D = (D_1, D_2, \dots, D_p)$ in which $D_j f(x) = \partial f(x)/\partial x_j$. Next, let $P(D)$ be a partial differential operator with constant coefficients which can be factored over the complex field into a product of linear combinations of the D_j. We will be concerned with a class of higher order Cauchy problems that includes the following

$$\begin{aligned} &\frac{\partial^n}{\partial t^n} w(x,t) = P(D)w(x,t), \\ &\frac{\partial^j}{\partial t^j} w(x,0) = \phi_j(x), \\ &\qquad j = 0, 1, \dots, n-1 \end{aligned} \tag{1.1}$$

as well as ones in which the underlying equation is a higher order generalization of the Euler Poisson Darboux equation. The primary objectives of this paper are: (a) to develop representations of solutions of these problems as multiple integrals of complex translations of the

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