ROCKY MOUNTAIN
JOURNAL OF MATHEMATICS
Volume 27, Number 3, Summer 1997

DISCONJUGACY AND TRANSFORMATIONS FOR SYMPLECTIC SYSTEMS

MARTIN BOHNER AND ONDŘEJ DOŠLÝ

ABSTRACT. We examine transformations and disconjugacy for general symplectic systems which include as special cases linear Hamiltonian difference systems and Sturm-Liouville difference equations of higher order. We give a Reid roundabout theorem for these systems and also for reciprocal symplectic systems. Particularly, we investigate a connection between eventual disconjugacy of linear Hamiltonian difference systems and their reciprocals. Finally, we present a disconjugacy-preserving transformation of a Sturm-Liouville equation of higher order which transforms this equation into another one of the same order.

1. Introduction. It has taken considerable effort to define disconjugacy for Sturm-Liouville difference equations of higher order

$$\text{(SL)} \qquad \sum_{\nu=0}^{n}(-1)^\nu \Delta^\nu \{r_k^{(\nu)} \Delta^\nu y_{k+n-\nu}\} = 0, \quad 0 \le k \le N$$

and to prove a so-called Reid roundabout theorem which contains the statement that disconjugacy is equivalent to positive definiteness of a certain related discrete quadratic functional. Recently, this problem was solved in [**10**] by treating (SL) as a special case of a linear Hamiltonian difference system

$$\text{(H)} \qquad \begin{aligned} \Delta x_k &= A_k x_{k+1} + B_k u_k, \\ \Delta u_k &= -C_k x_{k+1} - A_k^T u_k, \\ &\quad 0 \le k \le N \end{aligned}$$

(A, B, and C being square matrices) and by proving a Reid roundabout theorem for such more general systems.

In this paper we present an extension of those results to symplectic systems

$$\text{(S)} \qquad z_{k+1} = S_k z_k, \quad 0 \le k \le N$$

Received by the editors in revised form on February 10, 1997.