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A HIERARCHY OF INTEGRAL OPERATORS

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1. Introduction. The complex form of the Gauss theorem leads to a representation formula for complex functions w in $W^{1,p}(\mathcal{D})$ for bounded domains $\mathcal{D}$ with smooth boundary. This formula generalizing the Cauchy formula for analytic functions was proved by Pompeiu [**12**] and is called the Cauchy-Pompeiu formula. The area integral appearing in this formula defines a weakly singular integral operator T which plays an important role in the theory of generalized analytic functions as well as in the study of Beltrami and generalized Beltrami equations. Its properties were extensively studied by I.N. Vekua [**15**]. If the density ρ of this integral belongs to $L_p(\mathcal{D})$ with $2 < p$, then the integral $T\rho$ has first order weak derivatives $\partial(T\rho)/\partial\bar{z} = \rho$ and $\partial(T\rho)/\partial z =: \Pi\rho$, where $\Pi\rho$ is a singular integral understood as a Cauchy principal value. Integrals of this type even in higher dimensions were investigated by Calderon and Zygmund [**6, 7**]. Because the Π operator for the whole complex plane $\mathbf{C}$ turns out to be unitary in $L_2(\mathbf{C})$, the Riesz theorem [**13**] describes some important properties of this operator.

Many papers dealing with complex first order partial differential equations are based on properties of the T and Π operators; see, for example, [**3, 4, 5, 10, 16, 17**]. Recently second order complex equations have been investigated by means of integral operators which originate from the T operator by integration; see [**2, 8, 9, 10**]. In the paper [**18**] a complex fourth order equation is handled with an integral operator which can be connected with the T operator by repeated integrations of the latter.

In this paper these ideas are carried further to produce a hierarchy of integral operators $T_{m,n}$, defined for pairs of integers (m,n) with $0 \le m+n$, acting on certain $L_p(\mathcal{D})$ function spaces; the operator $T_{0,1}$ is the mentioned T operator while $T_{-1,1}$ is the Π operator and $T_{0,0}$ the identity operator. Whenever $0 < m+n$ the operators $T_{m,n}$ are regular or weakly singular, but for $m+n=0$ they are singular operators with properties analogous to those of the Π operator. Dzhuraev [**8**]

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