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# KUZNETSOV FORMULAS FOR GENERALIZED KLOOSTERMAN SUMS

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**1. Introduction.** Given two nonzero integers $m$ and $n$ and a positive integer $c$, the classical Kloosterman sum is defined as

$$K(m,n;c) = \sum_{\substack{1\le a\le c,\\ (a,c)=1}} e^{2\pi i(am+\bar{a}n)/c}$$

where $a\bar{a} \equiv 1 \pmod{c}$. We can also define a generalized Kloosterman sum

$$K_k(m,n;c) = \sum_{\substack{1\le a\le c,\\ (a,c)=1}} \varepsilon_a^{-\kappa}\left(\frac{c}{a}\right) e^{2\pi i(am+\bar{a}n)/c}$$

for odd $\kappa$ with $\kappa = 2k$, where $\varepsilon_a = 1$ if $a \equiv 1 \pmod{4}$ and $= i$ if $a \equiv 3 \pmod{4}$. Here $(c/a)$ is the extended Kronecker's symbol (Shimura [**16**] or see Iwaniec [**8**] and Sarnak [**14**]). In other words the generalized Kloosterman sum is the classical sum twisted by a character. It is known (Iwaniec [**8**]) that this generalized Kloosterman sum is essentially a Salié sum which is defined as

$$S(m,n;q) = \sum_{\substack{1\le a\le q,\\ (a,q)=1}} \left(\frac{a}{q}\right) e^{2\pi i(am+\bar{a}n)/q}$$

for odd integer $q$.

The Linnik-Selberg conjecture (Linnik [**13**], Selberg [**15**]) predicts that there is considerable cancellation in a weighted sum of the classical Kloosterman sums:

$$\sum_{1\le c\le x} \frac{K(m,n;c)}{c} = O(x^{\varepsilon})$$

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