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A SIMPLE PROOF OF FIEDLER'S CONJECTURE CONCERNING ORTHOGONAL MATRICES

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ABSTRACT. We give a simple proof that an $n \times n$ orthogonal matrix with $n \geq 2$ which cannot be written as a direct sum has at least $4n-4$ nonzero entries.

1. The result. What is the least number of nonzero entries in a real orthogonal matrix of order n? Since the identity matrix I_n is orthogonal the answer is clearly n. A more interesting question is: what is the least number of nonzero entries in a real orthogonal matrix which, no matter how its rows and columns are permuted, cannot be written as a direct sum of (orthogonal) matrices? Examples of orthogonal matrices of each order $n \geq 2$ which cannot be written as a direct sum and which have $4n-4$ nonzero entries are given in [**1**]. M. Fiedler conjectured that an orthogonal matrix of order $n \geq 2$ which cannot be written as a direct sum has at least $4n-4$ nonzero entries.

Using a combinatorial property of orthogonal matrices, Fiedler's conjecture was proven in [**1**]. A $(0,1)$-matrix A of order n is *combinatorially orthogonal* provided no pair of rows of A has inner product 1 and no pair of columns of A has inner product 1. Clearly, if Q is an orthogonal matrix of order n, then the $(0,1)$-matrix obtained from Q by replacing each of its nonzero entries by a 1 is combinatorially orthogonal. A quite lengthy and complex combinatorial argument is used in [**1**] to show that if A is a combinatorially orthogonal matrix of order $n \geq 2$ and A cannot be written as a direct sum, then A has at least $4n-4$ nonzero entries. Clearly this result implies Fiedler's conjecture. In this note we give a simple matrix theoretic proof of Fiedler's conjecture.

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