

ON THE STRUCTURE OF ROSENTHAL'S SPACE  $X_\varphi$   
IN ORLICZ FUNCTION SPACES

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ABSTRACT. Several kinds of complemented subspaces of Orlicz function spaces  $L^\varphi[0, 1]$  are studied. In particular  $X_\varphi$ , a natural generalization of Rosenthal's spaces  $X_p$ ,  $1 \leq p \leq \infty$ , is analyzed. Several isomorphic and structural properties of these spaces  $X_\varphi$  are given.

**0. Introduction.** Given an Orlicz function space  $L^\varphi[0, 1]$ , what do the complemented subspaces look like? In the particular case of  $L^p[0, 1]$  spaces,  $1 < p < \infty$ , Lindenstrauss and Rosenthal [10] have given a characterization of their complemented subspaces in terms of  $\mathcal{L}_p$ -spaces. But later it was shown that there exist at least uncountable many mutually nonisomorphic  $\mathcal{L}_p$ -spaces,  $1 < p \neq 2 < \infty$  [3].

In view of the above, it appears improbable that a complete classification of complemented subspaces of  $L^\varphi[0, 1]$  spaces will be obtained. For this reason, we limit ourselves to study here of several remarkable kinds of complemented subspaces of reflexive  $L^\varphi[0, 1]$  spaces. Such spaces will be defined in Section 2, the spaces  $X_\varphi$  and  $l^\varphi(w)(l_2)$ . The space  $X_\varphi$  was introduced in [18] as a generalization of Rosenthal's space  $X_p$ . The space  $X_p$ ,  $1 < p < \infty$ , was the first example of a complemented subspace of  $L^p[0, 1]$  nonisomorphic to the trivial subspaces  $l_2, l_p, l_2 \oplus l_p, L^p[0, 1]$  or  $(l_2 \oplus l_2 \oplus \dots)_p$ . The space  $X_p$  has interesting properties which have been studied in [17, 9, 1].

In [18], the space  $X_\varphi$  has been studied in relation with the structure of  $L^\varphi[0, 1]$ , proving that every sequence of independent symmetric random variables in  $L^\varphi[0, 1]$  spans a subspace of  $L^\varphi[0, 1]$  isomorphically embedded in  $X_\varphi$ . Nevertheless, here, the structure of these kinds of complemented subspaces of  $L^\varphi[0, 1]$ , their isomorphic properties, and

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