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FINITE HOMOGENEOUS SPACES

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ABSTRACT. We characterize homogeneous topological spaces which contain a nonempty discrete open subset. As a consequence, we characterize and enumerate finite homogeneous topological spaces.

1. Introduction. Several authors studied finite topological spaces with some topological structures, for example, Warren [**6**] gave a formula for the number of nonhomeomorphic topologies on a finite set by means of T_0 identification spaces.

Mashhour, Abd El-Monsef and Farrag [**4**] studied the maximum cardinality for topologies on a finite set, and in [**3**] they studied the number of topologies on a finite set.

A topological space X is called a homogeneous space provided that if p and q are two points of X, then there exists a homeomorphism $h : X \to X$ such that $h(p) = q$. The concept of homogeneity was introduced by W. Sierpinski [**5**]. Several mathematicians have studied homogeneous spaces. One may consult [**2**] for this study.

Let X be any set. By τ_{dis} and τ_{ind} we mean the discrete and indiscrete topologies on X, respectively. $|X|$ denotes the cardinality of the set X. If n is a natural number, $\tau(n)$ will denote the number of positive divisors of n.

2. Finite homogeneous spaces. In this section we shall give a characterization of finite homogeneous spaces, and we shall study the number of finite homogeneous topological spaces (up to homeomorphism).

Let us start with the following definition, see [**1**, p. 70].

Definition 2.1. Let $\{(X_\alpha, \tau_\alpha) : \alpha \in \Lambda\}$ be a collection of topological spaces such that $X_\alpha \cap X_\beta = \varnothing$ for all $\alpha \neq \beta$. Let $X = \cup_{\alpha\in\Lambda} X_\alpha$ be

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