ROCKY MOUNTAIN
JOURNAL OF MATHEMATICS
Volume 27, Number 4, Fall 1997

INTEGRALLY CLOSED IDEALS AND TYPE SEQUENCES IN ONE-DIMENSIONAL LOCAL RINGS

MARCO D'ANNA AND DONATELLA DELFINO

0. Introduction. Let $(R,\mathfrak{m})$ be a one-dimensional, local, Noetherian domain. Let $\overline{R}$ be the integral closure of R in its quotient field. The conductor of R in $\overline{R}$ will be denoted by $\mathfrak{C}$, and the length function on R-modules by $\lambda(-)$. We also assume that R is analytically irreducible, that is, $\hat{R}$ is a domain, or equivalently $\overline{R}$ is a DVR and is a finite R-module. If $\mathfrak{n}$ is the maximal ideal of $\overline{R}$, we assume that $R/\mathfrak{m} \simeq \overline{R}/\mathfrak{n}$. To any such ring we can associate a numerical semigroup as follows. Let v denote the valuation of the quotient field K of R, $v(K) = \mathbf{Z} \cup \{\infty\}$, with valuation ring $\overline{R}$ and set $v(R) = \{v(x) \mid x \in R, x \neq 0\}$. As $\overline{R}$ is a DVR and a finite R-module, $\mathfrak{C} = r^{g+1}\overline{R}$, where $r\overline{R} = \mathfrak{n}$. Therefore, $v(R)$ is a numerical semigroup such that $|\mathbf{N} - v(R)| < \infty$. We have $v(R) = \{0 = s_0, s_1, \dots, S_{n-1}, s_n = g+1, \rightarrow\}$, where $0 = s_0 < s_1 < \cdots < s_{n-1} < s_n = g+1$, and the arrow indicates that any integer strictly greater than g is in $v(R)$. The integer g is the greatest integer not in $v(R)$ and is called the Frobenius number of R. Matsuoka [**7**] defines a chain of ideals $\mathfrak{Ui}$ as follows

$$\mathfrak{Ui} = \{x \in R \mid v(x) \geq s_i\} \quad \text{if } i \leq n.$$

Clearly $\mathfrak{C} = \mathfrak{U}_n \subset \mathfrak{U}_{n-1} \subset \cdots \subset \mathfrak{U}_1 = m \subset R \subset \mathfrak{U}_1^{-1} \subset \cdots \subset \mathfrak{U}_{n-1}^{-1} \subset \mathfrak{U}_n^{-1} = \overline{R}$. Since $\lambda(\mathfrak{U}_{i-1}/\mathfrak{U}_i) = |v(\mathfrak{U}_{i-1}) - v(\mathfrak{U}_i)| = 1$ for all i, cf. [**7**], $n = |v(R) \cap \{0,1,\dots,g\}| = \lambda(R/\mathfrak{C})$. $\mathfrak{U}_i^{-1}$ is a ring for all i. Moreover, as $\overline{R}$ is local and finite over $\mathfrak{U}_i^{-1}$, $\mathfrak{U}_i^{-1}$ is a local ring. The sequence $t_i(R) = \lambda(\mathfrak{U}_i^{-1}/\mathfrak{U}_{i-1}^{-1})$ is called the *type sequence* of R (this terminology was first introduced in [**2**]). The name "type sequence" is related to the fact that, if $i = 1$, then $t_1(R) = \lambda(\mathfrak{m}^{-1}/R)$ is the Cohen-Macaulay type of R.

One can start with a numerical semigroup and define the analog of the notion of type sequence as follows. If $S = \{0 = s_0, s_1, \dots, s_n, \rightarrow\}$

Received by the editors on October 10, 1995, and in revised form on November 22, 1995.