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ON STRONGLY EXTREME POINTS IN KÖTHE-BOCHNER SPACES

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ABSTRACT. Let $E(X)$ be a Köthe-Bochner space and f an element of the unit sphere of $E(X)$. Then for f to be a strongly extreme point of the unit ball of $E(X)$ it is necessary that $\|f(t)\|_X$ be a strongly extreme point of E and that $f(t)/\|f(t)\|_X$ be a strongly extreme point of X for μ almost everywhere $t \in \operatorname{supp} f$. Furthermore, if E is order continuous, then the condition is also sufficient. If E is a nonorder continuous Orlicz space, then the unit ball of $E(X)$ has no strongly extreme points which gives a negative answer to the question about the criteria for the denting points of Köthe-Bochner spaces raised by C. Castaing and R. Pluciennik.

1. Introduction. Let (T, Σ, μ) be a measure space with complete σ-finite measure μ and L^0 the space of all (equivalence classes of) μ-measurable real valued functions. For $f, g \in L^0$, $f \leq g$ means $f(t) \leq g(t)$ for μ almost everywhere $t \in T$.

A Banach subspace E of L^0 is said to be a Köthe function space, if

(i) for any $f, g \in L^0$, $|f| \leq |g|$ and $g \in E$ imply $f \in E$ and $\|f\|_E \leq \|g\|_E$;

(ii) $\operatorname{supp} E = \cup\{\operatorname{supp} f : f \in E\} = T$.

A Köthe space E is said to be order continuous provided that $x_n \downarrow 0$ implies $\|x_n\| \to 0$.

If E is a Köthe function space over (T, Σ, μ) and X is a Banach space, then by $E(X)$ we denote the Köthe-Bochner Banach space of all (equivalence classes of) strongly measurable functions $f : T \to X$ such that $\|f(\cdot)\|_X \in E$ equipped with the norm $\|f\|_{E(X)} = \|\|f(\cdot)\|_X\|_E$.

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