ROCKY MOUNTAIN
JOURNAL OF MATHEMATICS
Volume 28, Number 1, Spring 1998

TWIN SEXTIC ALGEBRAS

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0. Introduction. Among all the symmetric groups, only S_6 has a nontrivial outer automorphism group, it being $\mathrm{Out}\,(S_6) = \{1, t\}$. Because of the exceptional involution t, a sextic separable algebra K over a ground field F has a "twin" K^t, well-defined up to unique isomorphism. The double twin K^{tt} is canonically isomorphic to the given algebra K. It may very well happen that K is a field but its twin K^t is not.

This paper presents a large number of facts, very useful to have at hand when one is investigating sextic algebras over a given ground field. We review the exceptional twinning operation in some detail. Also in our discussion of other topics, we continue to emphasize the helpful role played by this operation. The results presented in this paper are used in [**11**] for p-adic ground fields $F = \mathbf{Q}_p$ and in [**7**] for the rational ground field $F = \mathbf{Q}$.

Section 1 reviews the categorical approach to Galois theory; it is essential to take this approach because we must deal with algebras, not just fields. Section 2 centers upon simply giving the classical lists of possible Galois groups of sextic algebras: there are 40 intransitive groups, corresponding to nonfields, and 16 transitive groups, corresponding to fields. Here, in an effort to render the list as intelligible as possible, we organize the intransitive groups according to their "product closures."

Section 3 presents group-theoretic facts about the exceptional outer automorphism t of S_6. Then it discusses a basic formula due to Girstmair [**4**]; this formula explicitly associates to a polynomial f with $K = F[x]/f$ a new polynomial f^t which, if separable, satisfies $K^t = F[x]/f^t$. For comparison we also give a new much simpler twinning formula for an important special case.

Section 4 tabulates a wealth of explicit facts about the conjugacy classes and characters of the 56 groups in question; here the role played by t is emphasized constantly. Finally in Section 5 we give a geometric

Received by the editors on December 11, 1995.