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FRACTIONAL INTEGRALS OF IMAGINARY ORDER SUPPORTED ON CONVEX CURVES, AND THE DOUBLING PROPERTY

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1. Introduction and statement of results. Let $\Gamma : [0,\infty) \to \mathbf{R}^n$ be a curve in $\mathbf{R}^n$, $n \geq 2$, and define

$$H_\varepsilon f(x) = \int_0^\infty f(x-\Gamma(t)) \frac{dt}{(1+t^2)^{1/2+i\varepsilon}}$$

and

$$H_{\varepsilon,\delta} f(x) = \int_\delta^\infty f(x-\Gamma(t)) \frac{dt}{t^{1+i\varepsilon}},$$

for $x \in \mathbf{R}^n$, $f \in C_0^\infty(\mathbf{R}^n)$, $\varepsilon > 0$ and $\delta > 0$.

We seek conditions on Γ so that H_ε is a bounded linear operator on $L^2(\mathbf{R}^n)$ and the family of operators $\{H_{\varepsilon,\delta}\}$, for a fixed ε, is uniformly bounded on $L^2(\mathbf{R}^n)$.

The motivation for examining these operators is the work done by a number of researchers over the last 20 years in studying the L^p-boundedness of the Hilbert transform $\mathbf{H}_\Gamma$ and the maximal operator $\mathbf{M}_\Gamma$, defined for $x \in \mathbf{R}^n$ and $f \in C_0^\infty(\mathbf{R}^n)$ as follows

$$\mathbf{H}_\Gamma f(x) = \text{p.v.} \int_{-\infty}^\infty f(x-\Gamma(t)) \frac{dt}{t}$$

(a principle value integral), and

$$\mathbf{M}_\Gamma f(x) = \sup_{h>0} \frac{1}{h} \int_0^h |f(x-\Gamma(t))| \, dt.$$

Early inquiries into the L^p-boundedness of these operators, by Nagel, Rivière, Stein and Wainger, considered well-curved and two-sided homogeneous curves. A curve Γ in $\mathbf{R}^n$ is said to be well-curved if $\Gamma(0) = 0$

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