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LOGARITHMIC TRANSFORMATIONS INTO l^1

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ABSTRACT. Throughout this paper we shall write l to denote l^1. Let t be a sequence in $(0,1)$ that converges to 1, and define the logarithmic matrix L_t by $a_{nk} = -t_n^{k+1}/[(k+1)\log(1-t_n)]$. The matrix L_t determines a sequence-to-sequence variant of the logarithmic power series method of summability introduced by Borwein in [**1**]. The purpose of this paper is to study these transformations as mappings into l. A necessary and sufficient condition for L_t to be l–l is proved. The strength of L_t in the l–l setting is investigated. Also it is shown that L_t is translative in the l–l sense for certain sequences.

1. Introduction and background. Since the appearance of the famous Knopp-Lorentz theorem in [**5**], there have been many studies of the general properties of l–l summability methods, but still there are relatively few results about specific l–l methods. The shortage of examples of l–l methods and the study made by Fridy in [**3**] have provided the present study.

The logarithmic power series method of summability [**1**], denoted by L, is the following sequence-to-function transformation if

$$\lim_{x\to 1^-}\left\{\frac{-1}{\log(1-x)}\sum_{k=0}^{\infty}\frac{1}{k+1}u_k x^{k+1}\right\} = A,$$

then u is L-summable to A. In order to consider this method as a mapping into l, we must modify it into a sequence-to-sequence transformation. This is achieved by replacing the continuous parameter x with a sequence t such that $0 < t_n < 1$ for all n and $\lim_n t_n = 1$. Thus, the sequence u is transformed into the sequence $L_t u$ whose nth term is given by

$$(L_t u)_n = \frac{-1}{\log(1-t_n)}\sum_{k=0}^{\infty}\frac{1}{k+1}u_k t_n^{k+1}.$$

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