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HIGHER DIMENSIONAL AHLFORS REGULAR SETS AND CHORDARC CURVES IN $\mathbf{R}^n$

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1. Introduction. Recall that a Jordan curve C is *chordarc*, abbreviated CA, if there is a constant c such that for each pair of points $x, y \in C$ the arclength of one of the components A of $C\setminus\{x, y\}$ satisfies

$$l(A) \le c|x-y|.$$

Since $l(A) \ge \operatorname{diam}(A)$ for any arc A it is immediate that CA plane curves are quasicircles. (See Section 2 for many definitions and terminology.) Obviously CA curves are locally rectifiable. In fact, every CA curve is *Ahlfors regular*, abbreviated AR, which means that there is a constant b such that for all z and all $r > 0$ we have

$$l(C \cap B(z; r)) \le br.$$

It is folklore that Ahlfors regular quasicircles are chordarc. Another important property of CA curves, established by Tukia [**9**], and independently by Jerison and Kenig [**8**, 1.13], is that each one is bilipschitz equivalent to the circle $\mathbf{S}^1$ via a global homeomorphism of the plane. We summarize these comments as follows.

Theorem. *For a Jordan curve $C \subset \mathbf{R}^2$, the following are equivalent.*

(a) *C is chordarc.*

(b) *C is an Ahlfors regular quasicircle.*

(c) *$C = f(S^1)$ where $f : \mathbf{R}^2 \to \mathbf{R}^2$ is bilipschitz.*

Moreover, all constants depend only on each other and $\operatorname{diam}(C)$.

This paper is a product of our efforts to generalize the above theorem. We consider Jordan curves in $\mathbf{R}^n$ with Hausdorff dimension $\alpha \in [1, n)$.

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