

## FOURIER ANALYSIS ON COSET SPACES

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**ABSTRACT.** Let  $G$  be a locally compact group with a closed subgroup  $H$ . We will define and study natural analogs of the Fourier and Fourier-Stieltjes algebras for the homogeneous space  $G/H$  of left cosets of  $H$  in  $G$ . In particular, we show that when  $H$  is compact, the Fourier algebra  $A(G/H)$  of  $G/H$  can be used to study the nature of  $G/H$  in a manner similar to that of the group case.

**1. Introduction.** Let  $G$  be a locally compact group. Let  $B(G)$  be the Fourier-Stieltjes algebra of  $G$  as defined by P. Eymard in [6]. In a recent article, Bekka, Lau and Schlichting investigated the self-adjoint translation invariant subalgebras of  $B(G)$  [3]. In particular, they have characterized the self-adjoint two-sided translation invariant subalgebras of the Fourier algebra  $A(G)$  [3, Theorem 2.1]. They showed that these spaces could be identified as the functions in  $A(G)$  which are constant on cosets of some compact normal subgroup  $K$  of  $G$ . It follows that such algebras are isometrically isomorphic with the Fourier algebras of the quotient group  $G/K$ . Moreover, each compact normal subgroup determines a different subalgebra. It is an immediate consequence of this result that the structure of the quotient group  $G/K$  is reflected in algebra  $A(G)$ .

The result of Lau, Bekka and Schlichting can be viewed as a refinement of some earlier work of Takesaki and Tatsuuma [24]. In fact, Takesaki and Tatsuuma considered the left invariant self-adjoint subalgebras of  $A(G)$  and succeeded in establishing a one-to-one correspondence between such space and all compact subgroups  $K$  of  $G$ . In this case we are dealing with those functions which are constant on left cosets of  $K$ . However, when  $K$  is not normal, no link has been made between the nature of these subalgebras of  $A(G)$ , the structure of the homogeneous space  $G/K$  of left cosets of  $K$  and the structure of  $G$  itself. This is precisely the goal of this paper. We will give what we believe

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Received by the editors on August 1, 1995.

1991 AMS *Mathematics Subject Classification*. 43A85, 43A07, 43A15.

*Key words and phrases*. Locally compact group, cosets, Fourier algebra, amenable group.