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ON THE ABSOLUTE HARDY-BOHR CRITERIA

J. DEFRANZA, D.J. FLEMING AND B.E. RHOADES

ABSTRACT. We consider the general question of when the series to sequence variational summability domain, of an absolutely regular matrix method, is a sum space. In particular, for Nörlund polynomial methods it is shown to never be the case unless the method is equivalent to convergence.

1. Introduction. In most instances where the summability factors for a matrix method or pair of matrix methods have been determined, they have been characterized by the classical Hardy-Bohr conditions, see [**1, 3**]. In sequence space theory the notion of a sum space was introduced in [**13**], and in [**3**] it is shown that these classical conditions characterizing the summability factors are equivalent to the series to sequence convergence domain being a sum space. The situation is similar in the case of absolute summability factors, see [**2, 3**]. In [**6**] and [**7**], for example, certain classes of Nörlund methods are studied and results on when the series to sequence convergence domain is a sum space are given. Here we begin the study of similar questions in the context of absolute summability. In Section 2 we use the notion of a T-solid sequence space introduced in [**10**] to show that for any nontrivial Nörlund polynomial method N_p the summability domain $bv_{N_p\Sigma}$ is never a sum space. In Section 3 we note that, for an absolutely regular matrix method A, a necessary condnition for the series to sequence summability domain $bv_{A\Sigma}$ to be a sum space is that the method be of type $M(bv_0)$. This coincides with the situation for regular matrix methods given in [**3**]. That is, if $c_{A\Sigma}$ is a sum space, then the matrix A is of type M. In the final section we make several observations concerning certain absolutely regular Nörlund methods and pose the open question as to whether their series to sequence summability domains are in fact sum spaces.

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