ROCKY MOUNTAIN
JOURNAL OF MATHEMATICS
Volume 28, Number 1, Spring 1998

ARC APPROXIMATION PROPERTY AND CONFLUENCE OF INDUCED MAPPINGS

WŁODZIMIERZ J. CHARATONIK

ABSTRACT. We say that a continuum X has the arc approximation property if every subcontinuum K of X is the limit of a sequence of arcwise connected subcontinua of X all containing a fixed point of K. This property is applied to exhibit a class of continua Y such that confluence of a mapping $f : X \to Y$ implies confluence of the induced mappings $2^f : 2^X \to 2^Y$ and $C(f) : C(X) \to C(Y)$. The converse implications are studied and similar interrelations are considered for some other classes of mappings, related to confluent ones.

1. Introduction. For a metric continuum X we denote by 2^X and $C(X)$ the hyperspaces of all nonempty compact and of all nonempty compact connected subsets of X, respectively. Given a mapping $f : X \to Y$ between continua X and Y, we let $2^f : 2^X \to 2^Y$ and $C(f) : C(X) \to C(Y)$ to denote the corresponding induced mappings. Let $\mathfrak{M}$ stand for a class of mappings between continua. A general problem which is discussed in this paper is to find all possible interrelations between the following three statements:

$$f \in \mathfrak{M}; \tag{1.1}$$

$$2^f \in \mathfrak{M}; \tag{1.2}$$

$$C(f) \in \mathfrak{M}. \tag{1.3}$$

We do not intend to discuss the problem in full, for a wide spectrum of various classes $\mathfrak{M}$ of mappings. On the contrary, we concentrate our

Received by the editors on November 14, 1994, and in revised form on June 21, 1995.
1991 *Mathematics Subject Classification.* 54F15, 54B20, 54C10.
Key words and phrases. Approximation, arcwise connected, confluent, continuum, hyperspace, induced mapping, joining, pseudo-confluent, semi-confluent, weakly confluent, n-weakly confluent.