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SOME PROPERTIES OF NONLINEAR ADJOINT OPERATORS

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0. Introduction. In many branches of modern science and engineering nonlinear models more often are used. Nonlinear boundary value problems are studied to describe more precisely phenomena, for example, in the theory of plasticity, hydrodynamics, diffusion processes, biology, etc. In functional analysis this trend evokes the development of the nonlinear operator theory.

The present article is a slight contribution to this theory. Unlike the authors in [**1, 12–15, 17, 20, 21**], we consider a rather special class of operators from a Banach space into its dual involving nonlinearities of the power type. These operators, called polynomial and homogeneous operators, have some properties similar to linear operators. For example, for polynomial operators the continuity and the boundedness are equivalent. We generalize in a natural way some important notions known from linear analysis as the spectrum, numerical range, symmetry, self-adjointness and the normality. We show a number of their properties which can be useful for studying nonlinear operator equations, eigenvalue problems and other questions from nonlinear functional analysis and its applications.

1. Notations and definitions. Throughout this paper, let X, Y denote abstract (real or complex) Banach spaces and X^*, Y^* their dual spaces. By the symbol $\langle x^*, x\rangle$ we denote the value of a continuous linear functional $x^* \in X^*$ at a point $x \in X$. In case of Hilbert space X we use the same symbol for the inner product.

For the norm or weak convergence of the sequence $\{x_n\} \subset X$ to a point $x_0 \in X$ we use the symbols $x_n \to x_0$ or $x_n \overset{w}{\to} x_0$, respectively. Let $\mathbf{R}$ and $\mathbf{C}$ be the spaces of real and complex numbers, respectively. Further, we denote $S_1(0) = \{x \in X : \|x\| = 1\}$ the unit sphere in X.

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