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LIKE VANISHING HOLOMORPHIC RANDOM FUNCTIONS

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ABSTRACT. For every random function holomorphic in mean on an open connected subset D of $\mathbf{C}$ satisfying $\mathbf{P}[f(z) = 0] > 0$ for all $z \in D$, there is a measurable set $|\Delta$ satisfying $\mathbf{P}[\Delta] > 0$ and $f(z, \omega) = 0$ almost surely on Δ for every $z \in D$.

1. Introduction. The most realistic formulations of the equations arising in applied mathematics typically involve the study of random functions, which are presently a very active area of mathematical research (see [**1, 6, 10**]). On the other hand, it is of considerable interest in the stochastic analysis to know whether a sample property of a random function can be automatically derived from its behavior in mean [**2**–**4, 8**]. In [**8**] we proved that every random function holomorphic in mean on an open subset D of the complex field is equivalent to a random function whose paths are holomorphic on D. This paper is devoted to investigate the behavior of those random functions which are holomorphic in mean on an open connected subset D of $\mathbf{C}$ and vanish in a very broad sense; namely, for each $z \in D$, the event $[f(z) = 0]$ can happen, that is, each set $\Delta_z = \{\omega : f(z, \omega) = 0\}$ has a positive probability which depends on the element z. In such a case we prove that there is a measurable set Δ satisfying $\mathbf{P}(\Delta) > 0$ and $f(z, \omega) = 0$ almost surely on Δ for every $z \in D$. In particular, we obtain a surprising conclusion; namely, two holomorphic random functions f and g on D have versions with a nonzero probability of having common paths if, and only if, $\mathbf{P}[f(z) = g(z)] > 0$ for all $z \in D$.

2. The results. Throughout the paper, $(\Omega, \Sigma, \mathbf{P})$ will denote a complete probability space, and X will stand for a complex Banach space. Given a subset D of $\mathbf{C}$, a map $f : D \times \Omega \to X$ is said to be an *X-valued* (*first-order*) *random function* on D if, for each $z \in D$,

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