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HIGHER ORDER FLATNESS OF IMMERSED MANIFOLDS

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ABSTRACT. We prove that if a manifold M can be immersed into Euclidean space of codimension one, then the rth-order jet bundle $\mathrm{Jet}^r(TM)$ is flat for some $r \geq 0$. This is false if the codimension is greater than one: we give an example of a 4-manifold M^4 that immerses in $\mathbf{R}^6$ but for which none of the bundles $\mathrm{Jet}^r(M^4)$ is flat.

1. Introduction. The purpose of this note is to point out an interesting difference between manifolds that can be immersed in Euclidean space of codimension one and those that cannot. We show that manifolds that admit codimension one Euclidean immersions must satisfy a "higher-order flatness" condition not necessarily satisfied by other manifolds. The best way to describe this condition is in terms of the Andreotti invariant, which is defined as follows. Let $M = M^m$ be a C^∞ real manifold of dimension m. The *Andreotti invariant* $\mathcal{A}(M)$ is the smallest nonnegative integer r (if one exists) such that the rth order jet bundle

$$\mathrm{Jet}^r(M) := \mathrm{Jet}^r(TM) \cong TM \otimes S^r(TM \oplus 1),$$

where S^r is the rth symmetric power operator, admits a flat structure (i.e., admits an affine connection having curvature identically zero). If no such r exists, then we put $\mathcal{A}(M) = \infty$. The manifold M is said to be rth-order flat if $\mathrm{Jet}^r(M)$ is flat.

The Andreotti invariant and a similar invariant, the *alpha invariant*, of manifolds of constant positive curvature have been studied extensively using K-theory (see [**1, 4, 7, 8**]). These studies were motivated by earlier work, [**2, 3**] of Fredricks on the relationship between partial differential equations and higher-order differential geometry.

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