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EXCURSIONS OF A RANDOM WALK RELATED TO THE STRONG LAW OF LARGE NUMBERS

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1. Introduction. Let $\{X, i = 1, 2, 3, \dots\}$ be a sequence of independent and identically distributed random variables, each normally distributed with mean μ and variance σ^2. For $n = 1, 2, 3, \dots$, define $S_n = \sum_{i=1}^n X_i = S_0 \equiv 0$. It follows from the Kolmogorov strong law of large numbers (see [**1**, p. 274]), that $\lim_{n\to\infty}(S_n - n\mu)/n^\alpha = 0$ a.s. for all $\alpha > 1/2$. Consequently, for each real number $c > 0$, the inequality

$$S_n - n\mu > cn^\alpha \tag{1.1}$$

is satisfied for only finitely many indices n.

We define an excursion of the random walk $\{S_n, n = 1, 2, 3, \dots\}$ to be a complete sequence of consecutive indices for which the inequality (1.1) holds. More precisely, we say that an excursion of length k, $k = 1, 2, 3, \dots$, begins at index n, $n = 1, 2, 3, \dots$, if

$$(S_{n-1} - (n-1)\mu \leq c(n-1)^\alpha, S_{n+i-1} - (n+i-1)\mu > c(n+i-1)^\alpha \\ \text{for } i = 1, 2, 3, \dots, k, S_{n+k} - (n+k)\mu \leq c(n+k)^\alpha).$$

For $n = 1, 2, 3, \dots$, define the event A_n by $A_n = (S_n - n\mu > cn^\alpha, S_{n+1} - (n+1)\mu \leq c(n+1)^\alpha)$ and define the random variable $X(c)$ by

$$X(c) = \sum_{n=1}^\infty I(A_n). \tag{1.2}$$

($I(A)$ denotes the indicator function of the event A.) $X(c)$ represents the number of excursions. It follows from (1.1) that $X(c)$ is finite-valued. (We suppress, in the notation, the dependence of $X(c)$ on α.)

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