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RESONANCE FOR QUASILINEAR ELLIPTIC HIGHER ORDER PARTIAL DIFFERENTIAL EQUATIONS AT THE FIRST EIGENVALUE

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1. Introduction. In this paper the author presents a resonance result on the Sobolev space $W^{m,p}(\Omega)$ where Ω is a bounded open connected subset of R^N meeting the cone property. We let $1 < p < \infty$ and Qu be the $2m$th order quasilinear differential operator in generalized divergence form

$$Qu = \sum_{1\le|\alpha|\le m} (-1)^{|\alpha|} D^\alpha A_\alpha(x, \xi_m(u)), \tag{1.1}$$

for $u \in W^{m,p}$, where $\xi_m = \{D^\alpha u : 0 \le |\alpha| \le m\}$, and we make standard assumptions on A_α such as Carathéodory, uniform ellipticity, monotonicity, and a growth restriction. We shall study an equation of the following nature,

$$Qu(x) = g(x, u(x)) + h(x), \quad \text{for } u \in W^{m,p}(\Omega), \tag{1.2}$$

where $h(x) \in L^{p'}(\Omega)$, $p' = p/(p-1)$ and $g(x,t) : \Omega \times R \to R$ is Carathéodory. Subject to $mp > N$, we show the existence of a solution to (1.2) with g having superlinear growth in u but subject to a one-sided growth condition. Since Q lacks an $\alpha = 0$ order term, problem (1.2) is considered at resonance since $Qu = \lambda_1 u$ is solved by $\lambda_1 = 0$ and $u =$ constant, where λ_1 is defined as the first eigenvalue of Q. Shapiro [**9**, p. 365] provides a detailed explanation of this. This result primarily differs from that of Shapiro [**9**] in that our one-sided growth assumption on g is different from his, and since we approached the first eigenvalue of Q from values bigger than $\lambda_1 = 0$, in order for our results to hold, our Landesman-Lazer conditions must have reversed inequalities from those of Shapiro's theorem [**9**, p. 365]. Thus the theorem we will establish in this paper holds for a distinct class of functions that those meeting the hypothesis of Shapiro's Theorem 1. Examples meeting our conditions

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