ROCKY MOUNTAIN
JOURNAL OF MATHEMATICS
Volume 28, Number 3, Fall 1998

GLOBAL EXISTENCE FOR THE CAUCHY PROBLEM FOR THE VISCOUS SHALLOW WATER EQUATIONS

LINDA SUNDBYE

ABSTRACT. A global existence and uniqueness theorem of strong solutions for the initial-value problem for the viscous shallow water equations is established for small initial data and no forcing. Polynomial L^2 and L^∞ decay rates are established and the solution is shown to be classical for $t > 0$.

1. Introduction. The numerical solutions of the hyperbolic systems encountered in weather prediction models often develop high-frequency gravity wave solutions which can seriously distort short-term forecasts (on the order of hours to days; typically 12 hours). Various initialization schemes have been studied to control these distortions. The 'slow manifold,' proposed by Leith [**3**], is believed to have an invariance property such that if one begins with initial data on the slow manifold, the solution will remain free of gravity waves for all time. Temam [**11**] suggests a relationship between the mathematical concept of an inertial manifold and slow manifolds.

The shallow water equations are the simplest primitive equation model to exhibit gravity waves. However, before one can study the issues of global attractors and inertial manifolds, the question of global existence and uniqueness must be thoroughly addressed.

1.1. Well-posedness. Bui [**1**] proved local existence and uniqueness of classical solutions to the Dirichlet problem for the unforced viscous shallow water equations using Lagrangian coordinates and Hölder space estimates. He assumed the initial data $h_0 \in C^{1,\alpha}(\Omega)$ and $u_0 \in C^{2,\alpha}(\Omega)$.

Kloeden [**2**] proved global existence and uniqueness of classical solutions to the forced Dirichlet problem using Sobolev space estimates by following the energy method of Matsumura and Nishida [**5, 6**]. In addition to the assumptions 3–6 (Section 2), Kloeden further assumes the

Received by the editors on February 27, 1996, and in revised form on January 8, 1997.
AMS *Mathematics Subject Classification.* 34A05, 34A07.