

## APPROXIMATIONS OF UPPER SEMICONTINUOUS MAPS ON PARACOMPACT SPACES

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**ABSTRACT.** We prove theorems on graphic approximations of upper semi-continuous mappings which are natural analogues of Michael's selection theorems for lower semi-continuous mappings. Our convex-valued approximation theorem gives a generalization of Cellina's theorem in the sense that we omit the metrizability hypothesis. We also introduce a weakening of upper semi-continuity, the so-called quasi upper semi-continuity, and we show that approximation theorems are also valid for the class of quasi upper semi-continuous mappings. We obtain a finite-dimensional version of Kakutani's fixed-point theorem as a corollary of our finite-dimensional approximation theorem.

**1. Introduction.** In the theory of continuous selections of multi-valued lower semi-continuous maps, the key results are the following four theorems of E. Michael: the *convex-valued*, the *0-dimensional*, the *compact-valued* and the *finite-dimensional* selection theorem. Recall that a *selection* of a multi-valued map  $F : X \rightarrow Y$  is a (multi-valued) map  $G : X \rightarrow Y$  such that, for every  $x \in X$ ,  $G(x) \subset F(x)$ . The four theorems are summarized in Table 1.

In general, continuous selections do *not* exist for *upper* semi-continuous maps. Nevertheless, it makes sense to ask in this case about the existence of *approximations* of the given upper semi-continuous map  $F$  by a map whose graph is "close" to the graph of the map  $F$ . The following is known to be true [1–4], [12]:

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