CROSSED PRODUCT DUALITY FOR
PARTIAL C^*-AUTOMORPHISMS

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ABSTRACT. For partial automorphisms of C^*-algebras, Takai-Takesaki crossed product duality tends to fail, in proportion to the extent to which the partial automorphism is not an automorphism.

1. Introduction. Recently Exel [**1**] introduced the notion of a crossed product of a C^*-algebra by a partial automorphism (an isomorphism between ideals), in order to better understand circle actions. This generalizes crossed products by automorphisms (equivalently, integer actions), and some of the usual theory of crossed products by actions carries over to this new context [**1, 2, 5, 8**]. It seems natural to ask about the Takai-Takesaki crossed product duality [**9**]. In this paper we show that, perhaps unsurprisingly (since partial automorphisms, being partially defined, miss some of the information of the C^*-algebra), crossed product duality tends to fail for partial automorphisms. Indeed, crossed product duality seems to fail more miserably the more "partial" the partial automorphism is.

To be more precise, from experience with Takai-Takesaki duality for crossed products by abelian groups, we expect a dual action of the circle group $\mathbf{T}$ on a crossed product by a partial action, and indeed Exel [**1**] constructs such a thing. We apologize, but for our purposes we find it more convenient to work with the corresponding coaction of the integer group $\mathbf{Z}$. For abelian locally compact groups, statements about coactions are just Fourier transforms of statements about actions of the dual groups. However, at a certain point, we need a representation of the circle group $\mathbf{T}$, which is more easily dealt with as a representation of $c_0(\mathbf{Z})$. For the reader's convenience, in Section 2, after reviewing the elementary theory of partial automorphisms, we give a rough guide to