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A GENERALIZATION OF IFS WITH PROBABILITIES TO INFINITELY MANY MAPS

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ABSTRACT. This paper considers the problem of extending the notion of an IFS with probabilities from the case of finitely many maps in the IFS to the case of infinitely many maps. We prove that, under an average contractivity condition, the IFS is contractive in the Monge-Kantorovich metric. We also show that the invariant distribution is continuous with respect to the parameters of the IFS. Furthermore, using results of Lewellen, we obtain a result relating the support of the invariant measure to the attractor of the "geometric" IFS. Finally, we discuss the issue of the convergence of integrals with respect to the invariant measure and estimates on the error of these integrals.

1. Introduction. In his seminal paper [**3**], Hutchinson discusses the notion of self-similarity and introduces some ways to measure or define self-similarity. One such way is to say that a set $A \subset X$ is self-similar if there is come collection of maps $w_i : X \to X$ so that

$$A = \bigcup_i w_i(A).$$

In this way, A is seen to be made up of transformed copies of itself. Given this set of maps, one can define a set-valued map W by

$$W(B) = \bigcup_i w_i(B)$$

and we see that A is self-similar under the w_i's if A is a fixed point of W. While Hutchinson considered only finitely many maps, later Lewellen considered the case of infinitely many maps indexed by some compact metric space [**4**].

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