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SOME QUESTIONS AND CONJECTURES IN THE THEORY OF UNIVALENT FUNCTIONS

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ABSTRACT. The main object of this paper is first to answer a question of Campbell and Singh in the affirmative, and then to show that Komatu's conjecture and Thomas's conjecture are false at least in some cases.

1. Introduction. Let $\mathcal{S}$ denote the class of *normalized* analytic functions

$$f(z) = z + \sum_{n=2}^{\infty} a_n z^n,$$

which are univalent in the *open* unit disk $\mathcal{U}$. Also let $\mathcal{S}_R$ denote its subclass consisting of functions with real coefficients. The set of all odd functions in $\mathcal{S}$ is denoted by $\mathcal{S}^{(2)}$.

A function $f \in \mathcal{S}$ is said to be starlike of order α, denoted by $f \in \mathcal{S}^*(\alpha)$, if

$$\Re\left\{\frac{zf'(z)}{f(z)}\right\} > \alpha, \quad 0 \le \alpha < 1; \quad z \in \mathcal{U}.$$

Let

$$\mathcal{S}^*(0) = \mathcal{S}^* \quad \text{and} \quad \mathcal{K} = \{f : zf'(z) \in \mathcal{S}^*\}.$$

In this paper we first answer a question of Campbell and Singh [**1**] in the affirmative. We then show that Komatu's conjecture [**5**] and Thomas's conjecture [**7**, p. 166] are false at least in some cases.

2. A question of Campbell and Singh. Let

$$f(z) = z + \sum_{n=2}^{\infty} a_n z^n \in \mathcal{S}$$

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