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PROPERTY (u) IN $JH\tilde{\otimes}_\varepsilon JH$

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ABSTRACT. It is shown that the tensor product $JH\tilde{\otimes}_\varepsilon JH$ fails Pełczńyski's property (u). The proof uses a result of Kwapień and Pełczńyski on the main triangle projection in matrix spaces.

The Banach space JH constructed by Hagler [**1**] has a number of interesting properties. For instance, it is known that JH contains no isomorph of ℓ^1 and has property (S): every normalized weakly null sequence has a subsequence equivalent to the c_0-basis. This easily implies that JH is c_0-saturated, i.e., every infinite dimensional closed subspace contains an isomorph of c_0. In answer to a question raised originally in [**1**], Knaust and Odell [**2**] showed that every Banach space which has property (S) also has Pełczńyski's property (u). In [**4**], the author showed that the Banach space $JH\tilde{\otimes}_\varepsilon JH$ is c_0-saturated. It is thus natural to ask whether $JH\tilde{\otimes}_\varepsilon JH$ also has the related properties (S) and/or (u). In this note we show that $JH\tilde{\otimes}_\varepsilon JH$ fails property (u) (and hence property (S) as well). Our proof makes use of a result, due to Kwapień and Pełczńyski, that the main triangle projection is unbounded in certain matrix spaces.

We use standard Banach space notation as may be found in [**5**]. Recall that a series $\sum x_n$ in a Banach space E is called *weakly unconditionally Cauchy* (wuC) if there is a constant $K < \infty$ such that $\| \sum_{n=1}^k \varepsilon_n x_n\| \leq K$ for all choices of signs $\varepsilon_n = \pm 1$ and all $k \in \mathbf{N}$. A Banach space E has *property* (u) if whenever (x_n) is a weakly Cauchy sequence in E, there is a wuC series $\sum y_k$ in E such that $x_n - \sum_{k=1}^n y_k \to 0$ weakly as $n \to \infty$. If E and F are Banach spaces, and $L(E', F)$ is the space of all bounded linear operators from E' into F endowed with the operator norm, then the tensor product $E\tilde{\otimes}_\varepsilon F$ is the closed subspace of $L(E', F)$ generated by the weak*-weakly continuous operators of finite rank. In particular, for any $x \in E$ and $y \in F$, one obtains an element $x \otimes y \in E\tilde{\otimes}_\varepsilon F$ defined by $(x \otimes y)x' = \langle x, x'\rangle y$ for all $x' \in E'$.

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