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MULTIPLE POSITIVE SOLUTIONS FOR HIGHER ORDER BOUNDARY VALUE PROBLEMS

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ABSTRACT. Multiple positive solutions are shown to exist for the boundary value problem $u^{(n)} + f(t,u) = 0$, $\alpha u^{(n-2)}(0) - \beta u^{(n-1)}(0) = 0$, $\gamma u^{(n-2)}(1) + \delta u^{(n-1)}(1) = 0$, $u^{(i)}(0) = 0$, $0 \le i \le n-3$, when f is sublinear at one end point (zero or infinity) and superlinear at the other. The methods involve applications of a fixed point theorem for operators on a cone in a Banach space.

1. Introduction. In this paper we consider the two-point boundary value problem,

$$u^{(n)} + f(t,u) = 0, \quad 0 \le t \le 1, \tag{1}$$

$$\begin{aligned} \alpha u^{(n-2)}(0) - \beta u^{(n-1)}(0) &= 0, \\ \gamma u^{(n-2)}(1) + \delta u^{(n-1)}(1) &= 0, \\ u^{(i)}(0) = 0, \quad 0 \le i &\le n-3, \end{aligned} \tag{2}$$

where $f : [0,1] \times [0,+\infty) \to [0,+\infty)$ is continuous, $\alpha, \beta, \gamma, \delta \ge 0$, and $\rho = \beta\gamma + \alpha\gamma + \alpha\delta > 0$. Notice if $u(t)$ is a nonnegative solution of (1), (2), then $u^{(n-2)}(t)$ is concave on $[0,1]$.

When $n = 2$ the boundary value problem (1), (2), arises in nonlinear elliptical equations on an annulus, see [**2, 3, 11, 13, 14**]. In many physical and biological problems only positive solutions are of interest. Cones provide an elegant means to define positive elements in a Banach space. In [**4**] and [**5**] fixed point theorems with respect to a cone were used to find positive solutions for higher order boundary value problems. For a thorough treatment of cones in a Banach space, see Deimling [**6**] or Krasnosel'skii [**12**].

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