ROCKY MOUNTAIN
JOURNAL OF MATHEMATICS
Volume 28, Number 3, Fall 1998

SPHERE-FOLIATED CONSTANT MEAN CURVATURE SUBMANIFOLDS

WILLIAM C. JAGY

1. Introduction. In this article we will consider constant mean curvature submanifolds M of space forms ($\mathbf{R}^n$, $\mathbf{H}^n$ and $\mathbf{S}^n$). M will be of codimension one and will be 'foliated by spheres,' in a sense made precise below.

We begin with a few examples of constant mean curvature surfaces in $\mathbf{R}^3$ that are built up from *circles* in the sense we will be considering. The first is the catenoid, which is a minimal surface of revolution, given by the equation $r = \cosh z$ in cylindrical coordinates. The intersection of the catenoid with a plane $z = z_0$ is a circle with center on the z axis. The circles in two different such planes, say $z = z_1$ and $z = z_2$, are *coaxial*, meaning that the line joining their centers (the z axis) is orthogonal to both planes.

If we allow *nonzero* constant mean curvature, there are round spheres, such as $x^2+y^2+z^2 = 1$ in rectangular coordinates. Any family of planes (parallel or not) intersects the sphere in circles.

The well-known example that we shall call the "Riemann staircase" is also a complete minimal surface in $\mathbf{R}^3$. The intersections of a family of parallel planes (we take the planes $z = z_0$ again) with this surface are round circles, with the exception of a discrete set of straight lines. It is not a surface of revolution; if z_1 and z_2 are close together, the circles in planes $z = z_1$ and $z = z_2$ are not coaxial. As mentioned, for z near evenly spaced values z_j, the radius $r(z)$ goes to infinity as $z \to z_j$, and the plane $z = z_j$ intersects the surface in a straight line. There is a detailed discussion of this surface (and some lovely pictures) in an article by Hoffman and Meeks [**1**].

We are naturally led to ask whether the Riemann staircase is an isolated example—could something similar be found elsewhere, perhaps by allowing nonzero constant mean curvature, or by considering submanifolds of hyperbolic space?

Received by the editors on April 10, 1996.