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A KAPLANSKY THEOREM FOR JB*-ALGEBRAS

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ABSTRACT. We provide a new proof of a previously known result, namely every (not necessarily complete) algebra norm on a JB*-algebra generates a topology stronger than the one of the JB*-norm. As a consequence, if θ is a homomorphism of a JB*-algebra A into a Banach Jordan algebra B, then

(i) the range of θ is closed in B if θ is continuous,

(ii) θ is injective if and only if it is bounded below.

Introduction. A Jordan algebra is a nonassociative algebra A over the complex or real field in which the product satisfies $ab = ba$ and $(ab)a^2 = a(ba^2)$, $a, b \in A$. The Jordan triple product $\{abc\}$ is defined to be $(ab)c + a(bc) - (ac)b$, and for a in A, L_a denotes the operator of left multiplication by a.

A Banach Jordan algebra is a Jordan algebra A equipped with a complete norm $\|\cdot\|$, such that $\|ab\| \leq \|a\|\|b\|$, $a, b \in A$. A complex Banach Jordan algebra A with an involution $*$, such that $\|\{aa^*a\}\| = \|a\|^3$ for all a in A is called a JB*-algebra. It has been shown in [**18**] that in a JB*-algebra A the involution $*$ is an isometry, and every closed associative $*$-subalgebra of A is a C^*-algebra. this shows that the class of JB*-algebras coincides with the class of Jordan C^*-algebras introduced by Kaplansky in 1976, see [**17**]. For a JB*-algebra A, we denote by $C^*(a)$ the C^*-subalgebra of A generated by a self-adjoint element $a \in A$. If A is a C^*-algebra we define the Jordan product of two elements a, b in A by $a.b = (ab + ba)/2$. In terms of this product, A becomes a JB*-algebra. A closed linear $*$-subspace of a C^*-algebra B which is closed under the Jordan product is called a JC*-algebra. The theory of JB*-algebra is of capital importance in the theory of JB*-triples, and the classification of bounded symmetric domains in the complex Banach spaces, see [**6**] and [**9**].

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